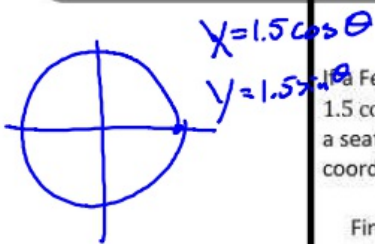


What you will learn about:
Patterns of Periodic Change



A Ferris wheel has a radius of 1.5 decimeters (15 meters), the function $1.5 \cos \theta$ and $1.5 \sin \theta$ give the x- and y-coordinates after rotation of θ for a seat that starts at the $(1.5, 0)$. Compare the graphs of these new coordinate functions with the graphs of the basic sine and cosine functions.

Find the maximum and minimum points of the graphs of $\cos \theta$ and $1.5 \cos \theta$ when:

$y = \cos x$	$y = 1.5 \cos x$	θ is measured in degrees	<table border="0"> <tr><td>max</td><td>$\cos \theta$</td><td>$1.5 \cos \theta$</td></tr> <tr><td></td><td>0</td><td>0</td></tr> <tr><td></td><td>360°</td><td>360°</td></tr> </table>	max	$\cos \theta$	$1.5 \cos \theta$		0	0		360°	360°	<table border="0"> <tr><td>min</td><td>$\cos \theta$</td><td>$1.5 \cos \theta$</td></tr> <tr><td></td><td>180</td><td>180</td></tr> <tr><td></td><td>π</td><td>π</td></tr> </table>	min	$\cos \theta$	$1.5 \cos \theta$		180	180		π	π
max	$\cos \theta$	$1.5 \cos \theta$																				
	0	0																				
	360°	360°																				
min	$\cos \theta$	$1.5 \cos \theta$																				
	180	180																				
	π	π																				
$y = \sin x$	$y = 1.5 \sin x$	θ is measured in radians	<table border="0"> <tr><td>max</td><td>$\cos \theta$</td><td>$1.5 \cos \theta$</td></tr> <tr><td></td><td>0</td><td>0</td></tr> <tr><td></td><td>2π</td><td>2π</td></tr> </table>	max	$\cos \theta$	$1.5 \cos \theta$		0	0		2π	2π										
max	$\cos \theta$	$1.5 \cos \theta$																				
	0	0																				
	2π	2π																				

Find the θ -axis intercepts of the graphs of $\cos \theta$ and $1.5 \cos \theta$ when:

θ is measured in degrees	<table border="0"> <tr><td>max</td><td>$\cos \theta$</td><td>$1.5 \cos \theta$</td></tr> <tr><td></td><td>$90^\circ, 270^\circ$</td><td>$90^\circ, 270^\circ$</td></tr> </table>	max	$\cos \theta$	$1.5 \cos \theta$		$90^\circ, 270^\circ$	$90^\circ, 270^\circ$
max	$\cos \theta$	$1.5 \cos \theta$					
	$90^\circ, 270^\circ$	$90^\circ, 270^\circ$					
θ is measured in radians	<table border="0"> <tr><td>max</td><td>$\cos \theta$</td><td>$1.5 \cos \theta$</td></tr> <tr><td></td><td>$\frac{\pi}{2}, \frac{3\pi}{2}$</td><td>$\frac{\pi}{2}, \frac{3\pi}{2}$</td></tr> </table>	max	$\cos \theta$	$1.5 \cos \theta$		$\frac{\pi}{2}, \frac{3\pi}{2}$	$\frac{\pi}{2}, \frac{3\pi}{2}$
max	$\cos \theta$	$1.5 \cos \theta$					
	$\frac{\pi}{2}, \frac{3\pi}{2}$	$\frac{\pi}{2}, \frac{3\pi}{2}$					

Find the maximum and minimum points of the graphs of $\sin \theta$ and $1.5 \sin \theta$ when:

θ is measured in degrees	<table border="0"> <tr><td>max</td><td>$\sin \theta$</td><td>$1.5 \sin \theta$</td></tr> <tr><td></td><td>90°</td><td>90°</td></tr> </table>	max	$\sin \theta$	$1.5 \sin \theta$		90°	90°	<table border="0"> <tr><td>min</td><td>$\sin \theta$</td><td>$1.5 \sin \theta$</td></tr> <tr><td></td><td>270°</td><td>270°</td></tr> </table>	min	$\sin \theta$	$1.5 \sin \theta$		270°	270°
max	$\sin \theta$	$1.5 \sin \theta$												
	90°	90°												
min	$\sin \theta$	$1.5 \sin \theta$												
	270°	270°												
θ is measured in radians	<table border="0"> <tr><td>max</td><td>$\sin \theta$</td><td>$1.5 \sin \theta$</td></tr> <tr><td></td><td>$\frac{\pi}{2}$</td><td>$\frac{\pi}{2}$</td></tr> </table>	max	$\sin \theta$	$1.5 \sin \theta$		$\frac{\pi}{2}$	$\frac{\pi}{2}$	<table border="0"> <tr><td>min</td><td>$\sin \theta$</td><td>$1.5 \sin \theta$</td></tr> <tr><td></td><td>$\frac{3\pi}{2}$</td><td>$\frac{3\pi}{2}$</td></tr> </table>	min	$\sin \theta$	$1.5 \sin \theta$		$\frac{3\pi}{2}$	$\frac{3\pi}{2}$
max	$\sin \theta$	$1.5 \sin \theta$												
	$\frac{\pi}{2}$	$\frac{\pi}{2}$												
min	$\sin \theta$	$1.5 \sin \theta$												
	$\frac{3\pi}{2}$	$\frac{3\pi}{2}$												

Find the θ -axis intercepts of the graphs of $\sin \theta$ and $1.5 \sin \theta$ when:

θ is measured in degrees	<table border="0"> <tr><td>max</td><td>$\sin \theta = 0$</td><td>$1.5 \sin \theta = 0$</td></tr> <tr><td></td><td>0, 180, 360</td><td>0, 180, 360</td></tr> </table>	max	$\sin \theta = 0$	$1.5 \sin \theta = 0$		0, 180, 360	0, 180, 360
max	$\sin \theta = 0$	$1.5 \sin \theta = 0$					
	0, 180, 360	0, 180, 360					
θ is measured in radians	<table border="0"> <tr><td>max</td><td>$\sin \theta = 0$</td><td>$1.5 \sin \theta = 0$</td></tr> <tr><td></td><td>0, π, 2π</td><td>0, π, 2π</td></tr> </table>	max	$\sin \theta = 0$	$1.5 \sin \theta = 0$		0, π , 2π	0, π , 2π
max	$\sin \theta = 0$	$1.5 \sin \theta = 0$					
	0, π , 2π	0, π , 2π					

How would the maximum and minimum points on the θ -axis intercept change if the Ferris wheel being modeled had radius a and the coordinate functions were $a \sin \theta$ and $a \cos \theta$?

$$y = 5 \cos \theta$$

max = 5	min = -5
0	180

$$y = 12 \sin \theta$$

max 12	min -12
90°	270°

$$y = 50 \cos \theta$$

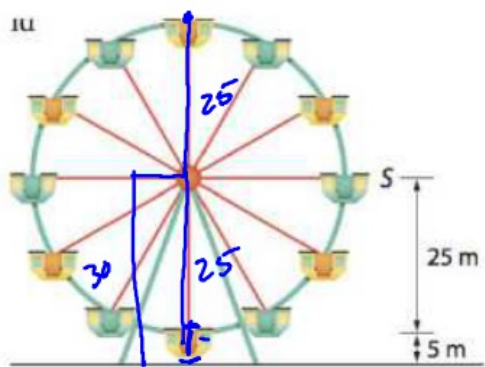
max = 50	min = -50
0	180

$$y = 1000 \sin \theta$$

max 1000	min -1000
90°	270°



When riding a Ferris wheel, customers are probably more nervous about their height above ground than their distance from the vertical axis of the wheel. Suppose a large Ferris wheel has a radius of 25 meters, the center of the wheel is located 30 meters above the ground, and the wheel starts in motion when seat S is at the "3 o'clock" position.



$$h(\theta) = \sin \theta$$

Modify the sine function to get a rule $h(\theta)$ that gives the height of seat S in meters after rotation of θ . Compare the graph of this height with the graph of $\sin \theta$.

$$h(\theta) = 25 \sin \theta + 30$$



Find the maximum and minimum points on the graph of $\sin \theta$ and $h(\theta)$ when:

	max 55	min 5
θ is measured in degrees	90	270°
θ is measured in radians	$\frac{\pi}{2}$	$\frac{3\pi}{2}$

Find the θ -axis intercepts on the graphs of $\sin \theta$ and $h(\theta)$

$$y = \sin \theta$$

$$0, 180, 360$$

$$h(\theta) = 25 \sin \theta + 30$$

$$\text{None}$$

How would the maximum and minimum points and the θ -axis intercept change if the Ferris wheel being modeled had a radius a and its center was c meters above the ground? Why is $c > a$?

$$y = a \sin \theta + c$$



$$y = A \sin Bt + c$$

A → Amplitude

Period → How long it takes to Repeat

$$\text{Per} = \frac{2\pi}{B}$$

Vertical Shift - c value

$$\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1}$$

Suppose that the height of a Ferris wheel seat changes in a pattern that can be modeled by the function $h(t) = 25 \sin t + 30$, where time is in minutes and height is in meters.

What are the period and amplitude of $h(t)$? What do those values tell about the motion of the Ferris wheel.

$$\text{Amp} = 25 - \text{Radius}$$

$$\text{Per} = \frac{2\pi}{B} = \frac{2\pi}{1} = 2\pi = 6.28 = 6 \text{ min } 17 \text{ sec}$$

If a seat starts out in the "3 o'clock" position, how long will it take the seat to return to that position? At what times will it revisit that position?

6 min 17 sec

12 min 34 sec

18 min 51 sec

Suppose the height (in meters) of seats on different Ferris wheels changes over time (in minutes) according to the functions give below. For each function:

- Find the height of the seat when the motion of the wheel begins
- Find the amplitude of $h(t)$. Explain what it tells you about motion of the wheel.

• Period • max/min height

$$h(t) = 15 \sin 0.5t + 17$$

$$h(t) = 24 \cos 2t + 27$$

$$\begin{aligned} \bullet h(0) &= 15 \sin .5(0) + 17 \\ &= 17 \text{ m} \end{aligned}$$

$$\bullet \text{Amp} = 15$$

$$\bullet P = \frac{2\pi}{.5} = 4\pi \quad 12 \text{ min } 33 \text{ sec}$$

$$\bullet \text{max } 32 \text{ min } 2$$

$$h(t) = 12 \sin 1.5t + 13$$

$$h(t) = -12 \cos t + 14$$

(.28)(60)